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The multidimensional solitons in a plasma: structure, stability and dynamics

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The formation, structure, stability and dynamics of multidimensional nonlinear waves and solitons in a plasma with $\mathbf{v}=-(\hat{\mathbf{z}}\times\nabla\psi)/B$ *and* $\beta>1$ *are studied. To study the stability of multidimensional solitons, the variation problem of the Hamiltonian bounding with respect to deformations conserving momentum is used. To study evolution of solitons and their collision dynamics the equations are integrated numerically. It was obtained that in both cases the formation of multidimensional solitons can be observed. It is found that the soliton elastic collisions can lead to formation of complex structures including the multisoliton bound states.*

1. Basic equations

In this paper, we study formation, structure, stability and dynamics of multidimensional solitons formed on the low-frequency branch of oscillations in a plasma for $\beta\equiv 4\pi nT/B^2 \ll 1$ and $\beta>1$. These oscillations are described by equation

$$\partial_t u + A(t,u)u = f, \quad f=\kappa\int_{-\infty}^{x}\Delta_\perp u\,dx, \qquad \Delta_\perp=\partial_y^2+\partial_z^2. \tag{1}$$

For $A_1(t,u)=\alpha u\partial_x-\partial_x^2(\nu-\beta\partial_x-\gamma\partial_x^3)$ Eq. (1) falls into GKP (Generalized Kadomtsev-Petviashvili) class of equations, and in the case when $\beta\equiv 4\pi nT/B^2\ll 1$ for $\omega<\omega_B=eB/Mc$, $k\lambda_D\ll 1$, describe propagation of the fast magnetosonic (FMS) wave in a magnetized plasma with $k_x^2\gg k_\perp^2$, $v_x\ll c_A$ near the cone of $\theta=\arctan(M/m)^{1/2}$ [1]. In this case, the function u is the dimensionless amplitude of the magnetic field of the wave $h=B_\sim/B$, the factors at the terms describing nonlinearity, dissipation and dispersion effects, respectively, are defined by plasma parameters and the angle $\theta=(\mathbf{B},\mathbf{k})$. In opposite case, $A_2(t,u)=3s|p|^2u^2\partial_x-$ $-\partial_x^2(i\lambda+\nu)$, Eq. (1) converts into 3D derivative nonlinear Schrödinger (3-DNLS) equation class and in the case when $\beta>1$ describes dynamics of the finite-amplitude Alfvén waves propagating nearly parallel to $\mathbf{B}$ for $u=h=(B_y+iB_z)/2B|1-\beta|$, $\mathbf{h}=\mathbf{B}_\perp/B_0$ where $p=(1+ie)$, and e is the "eccentricity" of the polarization ellipse of the Alfvén wave [2]. The upper and lower signs of $\lambda=\pm1$ correspond to the right and left circularly polarized wave, respectively; the sign of nonlinearity is accounted by the factor $s=\mathrm{sgn}(1-p)=\pm1$ in the nonlinear term; and $\kappa=-r_A/2$, $r_A=v_A/\omega_{0i}$.

Eq. (1) with A_1 or A_2 is not completely integrable. Therefore, excluding the stability and asymptotic analysis we used numerical integration for study of evolution of solitons and their collision dynamics using the special simulation codes.

2. Stability of 2D and 3D solutions

To study stability of the GKP equation solutions, we performed coordinate transformation and rewrite Eqs. (1,2) into the Hamiltonian form

$$\partial_t u=\partial_x(\delta H/\delta u), \tag{2}$$

where

$$H=\int\left[-\frac{\varepsilon}{2}(\partial_x u)^2+\frac{\lambda}{2}\left(\partial_x^2 u\right)^2+\frac{1}{2}(\nabla_\perp\partial_x v)^2-u^3\right]d\mathbf{r},$$

$\partial_x^2 v=u$, $\varepsilon=\beta|\gamma|^{-1/2}$, $\lambda=\mathrm{sgn}\,\gamma$. The stationary solutions of Eq. (2) are defined from the variation problem, $\delta(H+vP_x)=0$, where $P_x=\frac{1}{2}\int u^2 d\mathbf{r}$ is the momentum projection onto the x axis, v is the Lagrange's factor, illustrating the fact that all finite solutions of Eq. (2) are the stationary points of the Hamiltonian for fixed P_x. Thus, conforming with Lyapunov's theorem, it is needed to prove the Hamiltonian's boundedness (from below) for fixed P_x.
Let's consider in real vector space R the scale transformations $u(x,\mathbf{r}_\perp)\to\zeta^{-1/2}\eta^{(1-d)/2}u(x/\zeta,\mathbf{r}_\perp/\eta)$ (where d is the problem dimension, and $\zeta,\eta\in R$) conserving the momentum projection P_x. The Hamiltonian as a function of parameters ζ,η takes a form

$$H(\zeta,\eta)=a\zeta^{-2}+b\zeta^2\eta^{-2}-c\zeta^{-1/2}\eta^{(1-d)/2}+e\zeta^{-4} \tag{3}$$

where
$a=-(\varepsilon/2)\int(\partial_x u)^2 d\mathbf{r}$, $b=(1/2)\int(\nabla_\perp\partial_x v)^2 d\mathbf{r}$, $c=\int u^3 d\mathbf{r}$, $e=(\lambda/2)\int(\partial_x^2 u)^2 d\mathbf{r}$. In 2D case [$d$=2 in expression

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(3)] one can obtain that for $\lambda=1, \varepsilon \leq 0$ the Hamiltonian at fixed P_x is bounded from below, and, hence, the 2D solitons are absolutely stable. In cases $\lambda=1, \varepsilon>0$ and $\lambda=-1, \varepsilon<0$ H has local minima, and Eq. (2) may have the locally stable solutions for some parameters. All other cases correspond to unstable 2D solutions.

In 3D case we obtain that the absolutely stable 3D solutions take place for $\lambda=1, \varepsilon>0$, and the locally stable solutions can be observed for $\lambda=1, \varepsilon \leq 0$ if the condition $ab^2e/c^4<9/512$ is satisfied. The analysis to the problem of the FMS waves beam's propagation in magnetized plasma enables us to prove [1], for example, that the 3D beam propagating at θ angle to the magnetic field is not focusing and therefore becomes stationary and stable within the cone $\theta<\arctan(M/m)^{1/2}$ when $(m/M-\cot^2\theta)^2[\cot^4\theta(1+\cot^2\theta)]^{-1}>4/3$. We also note that obtained results give us the possibility to interpret correctly some numerical and theoretical results on the dynamics of the internal gravity wave solitons induced by the pulse-type sources in the F-region of the ionosphere [3].

To study stability of the 3-DNLS equation solutions we used the formal change $u \to h$ and investigated the boundedness of the Hamiltonian [2]

$$\mathrm{H}=\int_{-\infty}^{\infty}\left[\frac{1}{2}|h|^4+\lambda s h h^* \partial_x \varphi+\frac{1}{2}\kappa(\nabla_\perp \partial_x w)^2\right]d\mathbf{r}, \quad \partial_x^2 w=h,\ \varphi=\arg(h), \tag{4}$$

for transformation $h(x,\mathbf{r}_\perp)\to\zeta^{-1/2}\eta^{-1}h(x/\zeta,\mathbf{r}_\perp/\eta)$ $(\zeta,\eta\in\mathrm{C})$ conserving P_x, in complex vector space C. The Hamiltonian as a function of ζ,η is given by

$$\mathrm{H}(\zeta,\eta)=a\zeta^{-1}\eta^{-2}+b\zeta^{-1}+c\zeta^2\eta^{-2} \tag{5}$$

where $a=(1/2)\int|h|^4\,d\mathbf{r}$, $b=\lambda s\int hh^*\partial_x\varphi\,d\mathbf{r}$, $c=(\sigma/2)\int(\nabla_\perp\partial_x w)^2 d\mathbf{r}$. Solving the extremum problem for functional (5) we obtain that Hamiltonian (4) is bounded from below, i.e.,

$$\mathrm{H}>-3bd/(1+2d^2), \quad b<0 \tag{6}$$

if $ac^{-1}<d=\left(2\sqrt{2}\right)^{-1}\sqrt{13+\sqrt{185}}$, and in this case 3D solutions of 3-DNLS equation are stable. The solutions are unstable in the opposite case, $ac^{-1}\geq d,\ b<0$. Condition $b<0$ corresponds to the right circularly polarized wave with $p=4\pi nT/B^2>1$, i.e. when $\lambda=1,\ s=-1$, and to the left circularly polarized wave when $\lambda=-1,\ s=1$. It is necessary to note that the sign change $\lambda=1\to-1,\ s=-1\to1$ is equivalent to the change $t\to-t,\ \kappa\to-\kappa$ and for negative κ the Hamiltonian becomes negative in the area "occupied" by the 3D wave weakly limited in the $\mathbf{k}_\perp$-direction; in this case condition (6) is not satisfied. Change of sign of b to positive (when $\lambda=1,\ s=1$ or $\lambda=-1,\ s=-1$) is equivalent to the analytical extension of solution from real y, z to pure imaginary values: $y\to-iy,\ z\to-iz$ and, therefore, equivalent to the change of sign of κ in the basic equations. In this case instead of inequality (6) the opposite inequality will take place. From the physical point of view this means that if the opposite inequality is satisfied, the right polarized wave with the positive nonlinearity and the left polarized wave with the negative nonlinearity are stable.

3. Numerical results

In our numerical experiments we have investigated two types of the low-frequency oscillations in a plasma with $\beta<<1$ and $\beta>1$ which correspond to two types of waves and can lead to the formation of the multidimensional solitary wave structures. As a result, we have obtained that for FMS waves the 2D and 3D soliton formation can be observed. In particular, we observed the formation of a stable soliton with oscillating asymptotics, that corresponds to above mentioned analytical results. It is interesting to note that the 2D soliton interaction dynamics is not trivial for GKP equation unlike usual KP equation [4]. So, for example, for $\lambda=1,\ \varepsilon>0$ the formation of a stable two-soliton structure (so-called "bisoliton") can be observed as a final result of interaction of two initial pulses. In the 3D case for the FMS wave beam having the small angular distribution, the stationary propagation may be observed as a result of the nonlinear beam stabilization.

In the case of Alfvén waves propagating along the magnetic field lines, we have obtained that 3D stable solutions may be observed, with 3D spreading and collapsing ones. These results can be also interpreted in terms of the self-focusing phenomenon for the Alfvén waves' beam as the stationary beam formation, scattering, and self-focusing. Let's note that we observed the dynamics of the Alfvén waves' beam propagating in a plasma with $\beta>1$ at angles near 0° with respect to the magnetic field, and the dynamics of the FMS wave beam propagating in plasma with $\beta<<1$ at angles near $\pi/2$ with respect to the magnetic field. Let's note that for all cases the analysis of the Hamiltonian H deformations on the numerical solutions confirmed the stability of solutions considered above.

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